MAMMALS' BRAINS (PAGE 3)

Activity Time: 30–50 minutes

Required Document: Mammals.ftm

Fathom Prerequisites: Students should be able to

- Start Fathom
- Open a document

Fathom Skills: Students will learn how to

- Work with case tables
- Define an attribute with a formula
- Create graphs (dot plots, histograms, box plots)
- Select cases in a case table or graph
- Create a summary table of basic statistics

Mathematics Prerequisites: Students should be able to calculate the ratio of two quantities and find the median of a data set.

Mathematics Skills: Students will learn how to use one-variable graphs (dot plots, histograms, and box plots) to find particular values (such as greatest, least, and median) and use ratios to compare quantities. *Optional:* Students will explore bivariate data (see Extensions 3 and 5).

General Notes: At first, the mathematics in this activity may seem relatively simple—to identify particular values in a data set and to calculate ratios. However, you may be surprised that many students don't fully understand the need for ratios when comparing quantities of unequal size. For example, when a student looks only at the brain weights, she might say, "The African elephant has the largest brain." However, it's unfair to judge a mammal's brain size without considering the overall size of the mammal. When the same student looks at the ratios of brain weights to body weights, she may revise the conclusion to "The African elephant has the smallest brain."

The idea that the same data set can give two entirely different results is a subtle, yet important, aspect of this activity. The catch is to fully understand the definitions of "largest" and "smallest." Absolute brain weight and the ratio of brain weight to body weight are both acceptable ways to define brain size, albeit the latter is arguably more

meaningful. Seeing firsthand that the same data can be "manipulated" to give different conclusions will help students be careful and critical when they run across data in the media.

While comparing attributes and calculating ratios, Algebra 1 students will also get valuable practice in dimensional analysis. In the activity proper, students need to convert between grams and kilograms and understand what it means for a ratio to have no units. If you use Extension 2, students can also get exposure to derived units.

This activity is designed to be an easy first-time introduction to Fathom. The step-by-step worksheet leads students through many Fathom fundamentals, such as what an attribute is, how to create a case table, and how to make a graph. In order to save room, the activity does not give specific instructions for starting Fathom or opening a document; most students will have enough computer experience to accomplish these tasks without explicit instructions.

Although written primarily for Algebra 1, this activity can be successfully used for Algebra 2, Precalculus, or Statistics. First, if you have never used Fathom in these upper-level classes, you may want to use this activity as a gentle introduction to the software. Second, the concept of "largest" brain naturally leads to the question of intelligence, and the scientific study of animal intelligence provides an excellent opportunity for curve fitting and data transformation (see Extension 5).

MAKE A CONJECTURE

Q1 Possible answer: elephant, human, beaver, rat. Students might pick this order based on relative size of the mammals.

Q2 Possible answer: yes

Q3 Possible answer: human, elephant, rat, beaver. Students might pick this order by assuming that more intelligent mammals have larger brains compared to their overall body size.

INVESTIGATE

Q4 1320 g

Q5 0.28 kg. *Note:* Students will notice that the collection inspector and/or case table has some blank cells. This means that data were not available. Encourage students to leave these cells blank and to avoid the temptation to fill in meaningless data, such as zeros.

Q6 Most: African elephant, 5712 g; least: lesser short-tailed shrew, 0.14 g

Q7 African elephant, human, mountain beaver, rat

Q8 African elephant, human, mountain beaver, rat. Yes, this is the same order as for brain weights.

Q9 Students should use the dot farthest right in the graph of *BrainWeight:* African elephant, 5172 g.
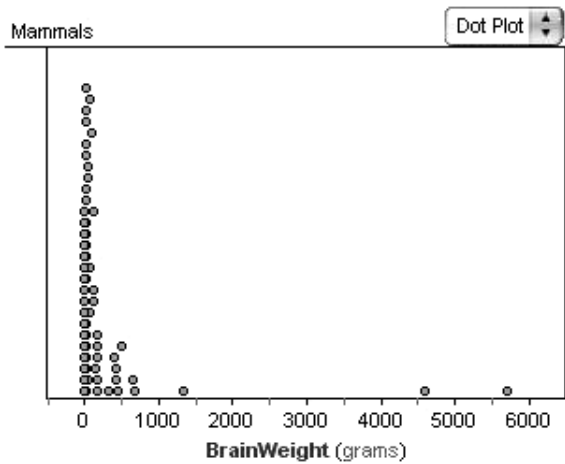
Q10 Because the dots are so densely clustered at 0 g, some students may have trouble picking out the least value. Other students may discover ways to adjust the axes and be able to zoom in enough to find the least value. The least value is still 0.14 g, the lesser short-tailed shrew.

Note: Even with the densely clustered dots, some students may be lucky enough to click at the bottom of the first stack of dots and find the lesser short-tailed shrew. This may lead them to erroneously believe that the stacks of dots always go least to greatest from bottom to top. This is not always the case. Actually, the dots are stacked relative to the cases' order in the case table. Students who find the lesser short-tailed shrew at the bottom of the stack

probably used **Sort Ascending** when answering Q6. As a counterexample, if students had last used **Sort Descending,** the lesser short-tailed shrew would have been at the top of the first stack of dots.

Q11 No, not all of the brain weights go in the same order as the body weights. For example, the third largest body weight belongs to the giraffe, which has the fourth largest brain weight, while the third largest brain weight belongs to the human, which has the thirteenth largest body weight.

Students could answer this by clicking dots in reverse order in the dot plot of *BodyWeight* and noticing that the dots in the dot plot of *BrainWeight* are not in the same order. Students could also sort the case table by *BrainWeight,* and then by *BodyWeight,* and notice that the species do not stay in the same order.

This means that some mammals have proportionately larger or smaller brains for their overall size. The brain-to-body ratios (coming up in steps 8–11) will account for brain size relative to the overall size of the mammal.

Q12 Giraffe (680 g) and horse (655 g). Students will probably click the bar between 500 and 1000, and see which cases are highlighted in the case table.
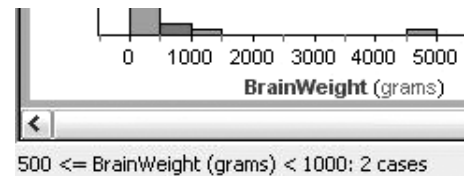
Note: If your students have formally learned about histograms, you should point out that the bin is officially $500 \le x < 1000$, which does not include the upper bound.

Q13 Possible answers: 6 g/kg, 0.006 g/g, 0.006 kg/kg, or 0.006 (no units). All four of these ratios are equivalent. Because both measurements are weights, it is best to convert to the same units (either grams or kilograms) before finding the ratio, and then write the ratio 0.006 without units.

Q14 0.006; technically there are no units. *Note:* For students who could find only the ratio 6 g/kg for Q13, you should explain how Fathom performs unit conversions on compatible units, resulting in a value that has no units.

Q15 Largest: ground squirrel, 0.0396; smallest: African elephant, 0.00086. No, these are not the mammals that had the brains that weigh the most and least; in fact, the African elephant is now on the opposite end of the spectrum.

Q16 Human, rat, mountain beaver, African elephant. This may not be the same order predicted in Q3.

Q17 It means, for example, that the human's brain is proportionately larger than the brains of the other mammals.

Q18 More mammals have brain-to-body ratios below 0.01. In the box plot, the median value falls below 0.01, so more than 50% of the mammals have brain-to-body ratios below 0.01.

Students could also create a histogram and adjust the bin widths to show that 41 cases have brain-to-body ratios below 0.01.

0 <= BrainToBody < 0.01000: 41 cases
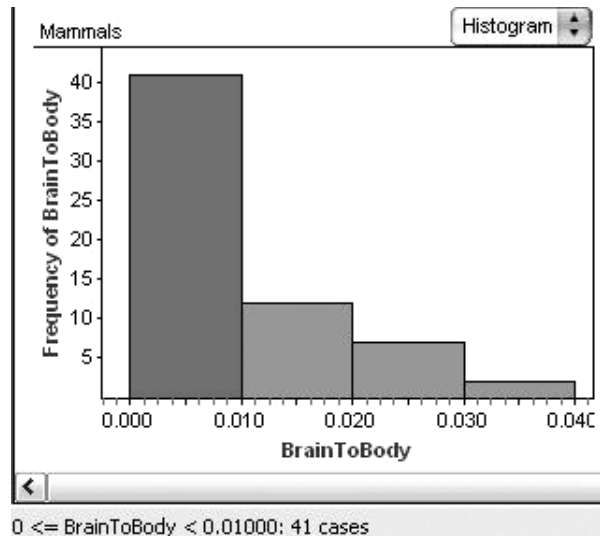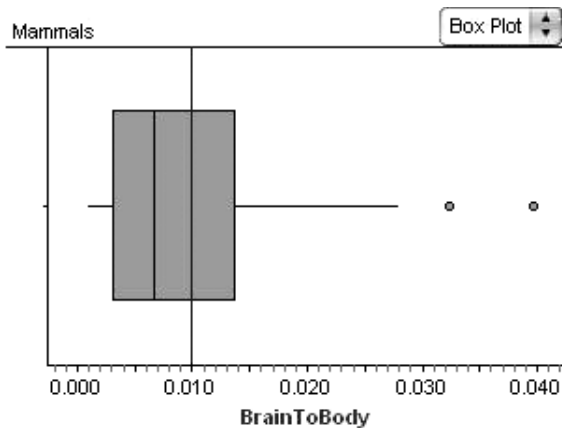
Q19 Possible answers: median: 0.0065; interquartile range: 0.0105

Q20 Median: 0.0066; interquartile range: 0.0108

Mammals	
BrainToBody	0.0096242136
	0.00085843102
	0.0030857143
	0.0066108564
	0.013842482
	0.03960396
	0.010756768

S1 = mean( )
S2 = min( )
S3 = Q1( )
S4 = median( )
S5 = Q3( )
S6 = max( )
S7 = iqr( )

Q21 Any answer with a reasonable explanation is acceptable. Some students might use the median or mean exactly as calculated in the summary table; they might try combining a measure of center with a measure of spread to give a range of values; they might use the distribution in a dot plot to make a subjective answer based on where the dots cluster; or they might use a histogram and pick out bins that have more values than others (essentially the mode). Some students may not give a value at all, preferring to argue that the data have so much spread that it is meaningless to state a "typical" value.

EXPLORE MORE

1. The formula for brain-to-body percentage is $100(BrainWeight/BodyWeight)$. Students should notice that the only difference between these values and the brain-to-body ratios is that the decimal point is moved two places to the right. Even the graphs will look the same except the axes will differ by a factor of 100. Students should realize that this makes mathematical sense because they've only multiplied the brain-to-body ratios by 100.

2. Students might say that neither brain weight nor brain-to-body ratio is a good way to judge intelligence. To support their answers, students might point to the position of "intelligent mammals" (such as human and chimpanzee) relative to less intelligent mammals. For example, if we use brain weight as a judge of intelligence, both species of elephants are more intelligent than humans, and the chimpanzee is on about the same level as a donkey. Using brain-to-body ratio as a judge of intelligence makes the ground squirrel the most intelligent mammal and there are seven other mammals more intelligent than humans.

EXTENSIONS

1. When making the box plot of *BrainToBody,* students will notice that two points appear at the end of the right whisker. Discuss why these two points may be excluded from the rest of the whisker. You may want to introduce the term *outlier.*

   For students who have already learned about outliers and different methods of calculating outliers, ask them to verify that these two points surpass the criterion *median* $\pm$ 1.5 *IQR.*

2. The data used in this activity come from a classic study of the sleep habits of 62 mammals (originally published in the article "Sleep in Mammals: Ecological and Constitutional Correlates" by Allison Truett and Domenic V. Cicchetti (1976), *Science,* November 12, vol. 194, pp. 732–734). Although the activity proper examines only brain weight and body weight, there are rich connections between the other attributes as well.

   Have students explore ratios of other attributes for these mammals, such as dreaming sleep to total

sleep, length of gestation to life span, or brain weight to length of gestation. Use the ratios to phrase and answer questions such as "Which mammal dreams the most?" or "Which mammal spends relatively the most time developing in the womb?"

Note: Depending on the attributes chosen, students may encounter situations when Fathom will create a "derived unit." For example, when comparing brain weight to length of gestation, the units (grams and days) cannot be converted into compatible units. Instead, the attribute *BrainToGestation* has the unit grams per day.

3. To extend this activity beyond one-variable statistics, have students make scatter plots for pairs of attributes, such as *BrainWeight* versus *BodyWeight.*
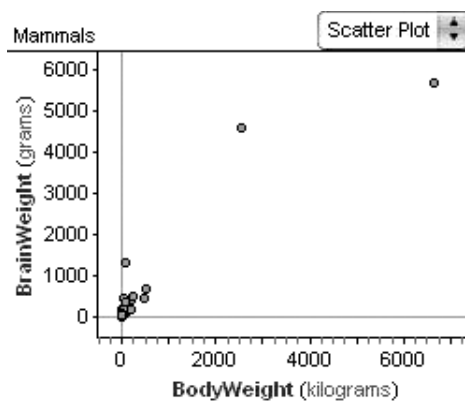
Some bivariate graphs show clear relationships; this one, in particular, implies that as body weight increases, brain weight also tends to increase. Students can try fitting a line or any other function to the graph. (See Extension 5 for an exponential function that models this relationship.) *LifeSpan* versus *Gestation, TotalSleep* versus *Gestation,* and any combination of the three sleep attributes also provide graphs that have recognizable patterns.

As a further extension, students could try comparing some of the categorical attributes (*Predation, Exposure,* and *Danger*) with the numerical attributes or with each other. They can find an explanation of these attributes by double-clicking the collection to show the inspector and then clicking the **Comments** tab.

4. Some students may be interested in researching the study of intelligence in different species and writing a short paper on the mathematics used in this field of science. Two key words that students should research are *allometry* (the study of the growth of one part of an organism in relation to the growth of the whole organism) and *encephalization* (the amount of brain mass relative to body size).

Students will learn that some scientists use exponential functions to estimate brain weight as a function of body weight. Then the *encephalization quotient*—the ratio of actual brain weight to estimated brain weight for an animal of that size—is used to judge relative intelligence. Animals that have an actual brain weight greater than the estimated brain weight (an encephalization quotient greater than 1) are considered more intelligent.

Students may also run across competing theories on intelligence, such as judging intelligence based on the number of neurons in the brain rather than on brain size.

5. Students in Algebra 2, Precalculus, or Statistics can perform log-log transformations to find an exponential model for predicting brain weight as a function of body weight. As mentioned in Extension 4, some scientists use exponential functions to calculate the ratio of actual brain weight to expected brain weight (the encephalization quotient), which is an indicator of relative intelligence.

First, create the attributes *logBody* and *logBrain* and create a scatter plot. (*Note:* Before taking logarithms, change the units for both *BodyWeight* and *BrainWeight* to grams or kilograms.) Use a least-squares line on the transformed data and work backward to derive an exponential function that fits the original data. Below, *logBrain* versus *logBody* is modeled by the least-squares line *logBrain* = 0.752 *logBody* − 1.3. This converts to the exponential function *BrainWeight* = 0.05 *BodyWeight*$^{0.752}$. (*Note:* In order to get appropriate units, enter the function as

```
BrainWeight = 0.05 scalar(BodyWeight)^0.752 g
```

Mammals' Brains, Extension 5

Mammals

	Species	BodyWe...	logBody	BrainWe...	logBrain	EstBrain	EQ
units		grams		grams		grams	
1	Human	62000 g	4.79239	1320 g	3.12057	200.839 g	6.57242
2	Rhesus monkey	6800 g	3.83251	179 g	2.25285	38.108 g	4.69717
3	Baboon	10550 g	4.02325	179.5 g	2.25406	53.022 g	3.38539
4	Owl monkey	480 g	2.68124	15.5 g	1.19033	5.19115 g	2.98585
5	Chimpanzee	52160 g	4.71734	440 g	2.64345	176.363 g	2.49485
6	Ground squirrel	101 g	2.00432	4 g	0.60206	1.60775 g	2.48794
7	Patas monkey	10000 g	4	115 g	2.0607	50.9296 g	2.25802

BrainWeight = 0.05 scalar (BodyWeight)$^{0.752}$g

logBrain = 0.752logBody − 1.3; r^2 = 0.92
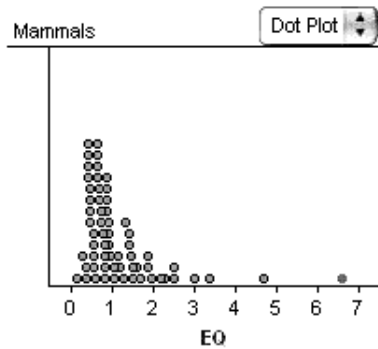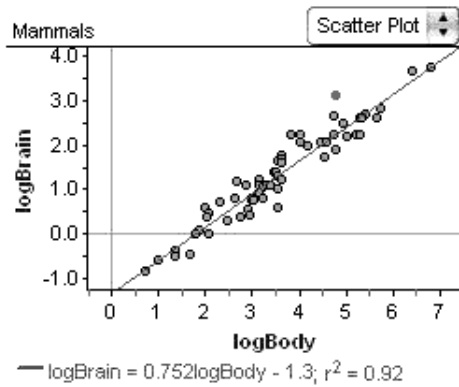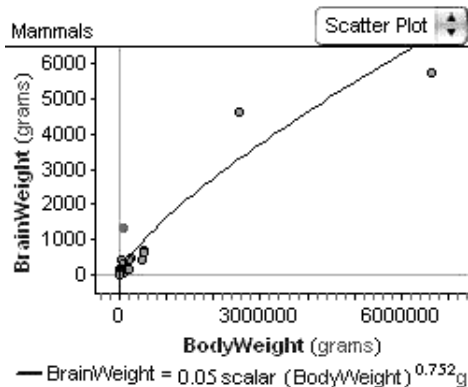
The function *scalar* removes the units from *BodyWeight* during the calculation, and then "g" reapplies grams. Otherwise, the units would be $g^{0.752}$.)

The same exponential function is used to define an attribute (*EstBrain*) that calculates the estimated brain weights, and then another attribute (*EQ*) calculates the encephalization quotient. Notice that *EQ* puts the mammals in an order that most people (and many scientists) find reasonable, with humans as the most intelligent, followed by many of the primates. *(See bottom of previous page.)*

READING THE NEWS (PAGE 7)

Activity Time: 50–80 minutes

Materials:

- One copy of the front section of a newspaper for each pair of students
- Centimeter rulers
- Clocks with a second hand or stopwatches

Fathom Prerequisites: None

Fathom Skills: Students will learn how to

- Create a collection by entering data into a case table
- Create attributes
- Create graphs (scatter plots)
- Graph lines of fit (movable lines)
- Plot a function
- Trace a function
- Add multiple attributes to an axis

Mathematics Prerequisites: Students should be able to make and understand a scatter plot of points and find the slope of a line by counting rise over run or using two points in the slope formula.

Mathematics Skills: Students will learn how to interpret slope as a rate or speed and compare slopes based on their graphical and numerical representations. Students will also find a direct-variation linear equation to model data that are roughly linear; identify the slope of a direct-variation linear equation as the coefficient of the independent variable; and use a slope or a linear equation to extrapolate data. *Optional:* Students will analyze one-variable statistics (see Extension 1).

General Notes: This activity is about slope and direct variation. Students consider the question "How long would it take to read the front section of the newspaper aloud?" First, students collect data and do an analysis offline, primarily using slope to extrapolate. Then students use Fathom to do further analysis, extrapolating with linear equations. Because the data will be very close to a direct

variation, this is an excellent opportunity for Algebra 1 students to practice slope and linear equations while learning the basics of statistical linear modeling.

Because the activity requires collecting data, by-hand analysis, and then Fathom, you might plan to use the activity over two or more days. For example, you could do data collection and by-hand analysis on the first day, and then go to the computer lab and use Fathom on the second day.

While collecting data, students might encounter many interesting sources of variability. Here are some issues for you and your students to consider:

- This is not a race. Students should read as if they are radio announcers.
- Every reader should read new material each time, because rereading is faster.
- Every reader should read from columns of the same width. Pictures don't count, but students will have to decide how to incorporate headlines, changes in font size, and other irregularities.
- Students vary in their ability to measure time accurately. You may want to measure it for them or let them practice beforehand.

When students make a by-hand scatter plot of their data, it is assumed that they can plot coordinate pairs, recognize a relationship in the bivariate data, draw a line to fit the data, and calculate the slope of the line. Depending on your students' experience with these concepts, you may need to offer guidance through steps 4–5 and questions Q5–Q9.

The slope in this activity represents a rate or speed—centimeters per second—and students are expected to use this rate to extend the relationship and extrapolate data. Because the slope is a ratio, you may want to compare this activity to the previous activity, Mammals' Brains, in which the brain-to-body ratios did not represent a constant rate that could be applied to all species. Additionally, those ratios had no units (grams over grams cancelled), whereas the ratio in this activity does have a derived unit (centimeters per second).

For brevity, the by-hand portion of the activity has students extrapolate only twice: once with the raw data (Q4) and once with the slope (Q9). You may prefer to stop

at these two questions, discuss their solutions with the entire class, and ask additional extrapolation questions. If you do, be sure to use an equal number of questions that require extrapolating given the independent value and given the dependent value.

The online part of the activity hopes to show students how technology can help make calculations easier. However, be careful not to undermine the importance of the by-hand methods. Students, especially those in Algebra 1, should have ample experience approximating lines of fit and finding the equation of a line both by hand and with technology.

After students finish the activity, and have had a lot of experience with their own and their partner's slope/reading speed, you may challenge them to extend their conceptual understanding. For example, ask, "What do you think is the steepest slope anyone could possibly have in this situation?," "Is it possible to have a slope of 0?," and "Is it possible to have a negative slope?"

Another good topic of discussion is whether or not students read at a constant rate. Ask, "Did you read at a constant rate?," "How do you know?," "How can you use the graph to identify when you read a little bit faster or a little bit slower?," and "Comparing your graph to your partner's graph, who read more closely to a constant rate?" Obviously, a constant rate would make the points perfectly linear, while a little faster or slower would make the points fall above or below the line of fit, respectively. Determining who read more closely to a constant rate can be done qualitatively or quantitatively; you could even extend the quantitative analysis into an introduction of residuals, residual plots, and residual squares.

Although written primarily for Algebra 1, this activity can be successfully used for Algebra 2, Precalculus, or Statistics. In addition to using a movable line, have students compare the results using median-median lines and/or least-squares lines. You can use these alternative models to discuss the danger of extrapolating too far beyond the domain and range of the known data. For example, have students answer Q13 for each of the different models and record how much the results vary. This will illustrate how small changes in the slope are magnified into big differences as you extrapolate farther away from the data. Extension 2

offers another idea that can help Algebra 2, Precalculus, and Statistics students better understand the sensitivity of the least-squares line.

MAKE A CONJECTURE

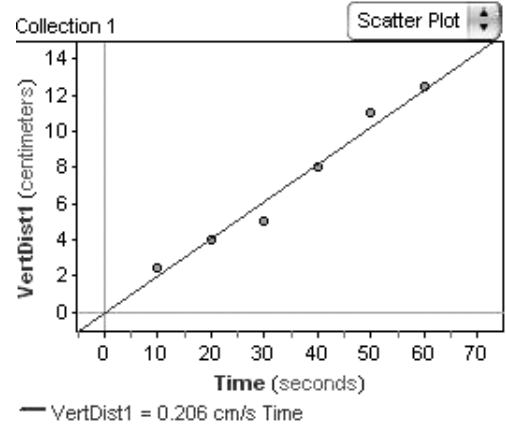Q1 Answers will vary.

Q2 Answers will vary.

INVESTIGATE (BY HAND)

Almost every answer for this activity will vary depending on your newspaper and the speed at which a student reads. The following answers give sample answers as a point of reference only.

Q3 A typical page might have 150 vertical centimeters of text. Students should pick a representative page and measure the total vertical distance of all columns.

Q4 A typical reading rate is 0.2 centimeter per second, so the table of data might show 2 centimeters in 10 seconds. If a typical page has 150 centimeters, students would need 150 ÷ 2, or 75 ten-second periods—750 seconds total—to read one page. A similar result could be calculated using any time-distance pair from the table of raw data.

Q5 Yes, the lines should go through (0, 0). If you have no time to read (0 seconds), you won't be able to read anything (0 centimeters).

Q6 A typical slope is 0.2 centimeter per second.

Q7 The slope is the average reading speed in centimeters per second.

Q8 Because the slope represents speed, the faster reader should have the steeper line, that is, the line with the greater slope.

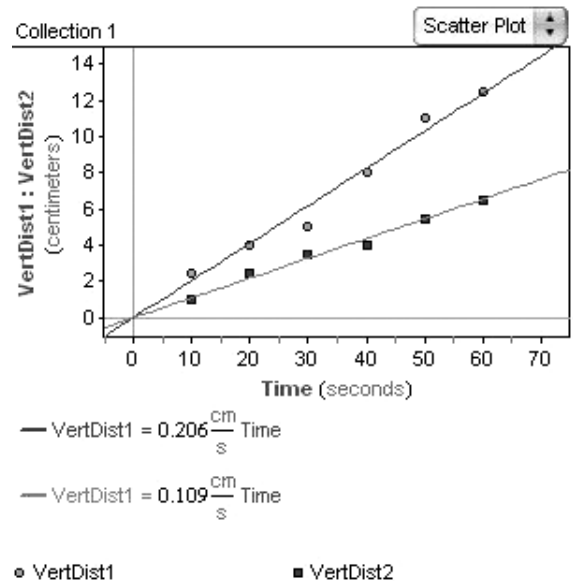Q9 Students need to multiply the slope by the time. For example, 0.2 cm/s • 100 s = 20 cm.

INVESTIGATE (WITH FATHOM)

Q10 Here's a typical graph with the linear equation $VertDist1 = 0.206$ cm/s $Time$.

Collection 1   Scatter Plot

(Scatter plot: VertDist1 (centimeters) on vertical axis 0–14, Time (seconds) on horizontal axis 0–70)

— VertDist1 = 0.206 cm/s Time

Q11 The coefficient of $Time$ represents the slope of the line and should be close to the slope calculated in Q6. (*Note:* You may need to define *coefficient.*)

Q12 Here's a sample dot plot for a student who reads about 0.2 cm/s and another who reads about 0.1 cm/s. The equations of the two lines are both in the form $VertDist1 = slope \cdot Time$, so you can identify the faster reader by looking at the coefficient of $Time$.

Collection 1   Scatter Plot

(Scatter plot: VertDist1 : VertDist2 (centimeters) on vertical axis 0–14, Time (seconds) on horizontal axis 0–70)

— $VertDist1 = 0.206 \frac{cm}{s} Time$

— $VertDist1 = 0.109 \frac{cm}{s} Time$

○ VertDist1   ■ VertDist2

Q13 The sample answers use the function and graph from Q10. Note that parts a and d require students to match a particular value of *Time* (the independent value) and read the coordinate for *VertDist* (the dependent value). Part b requires students to match *VertDist* and read *Time*. Students may need to reduce the size of the graph or adjust the axes in order to trace far enough.

a. (99.943 s, 20.588 cm), approximately 20.6 centimeters

b. (728.20 s, 150.01 cm), approximately 728 seconds, or 12 minutes. *Note:* This answer uses the 150 cm approximated in Q3.

c. If a typical front section is 8 pages, multiply answer b by 8; approximately 5826 seconds, or 97 minutes. *Note:* Students will probably have trouble tracing until *VertDist* is 1200 cm. That's why this solution uses multiplication.

d. (1802.0 s, 371.21 cm), approximately 371 centimeters, or 2.5 pages

Q14 Based on the answer to Q13c or d, students are likely to say, "No, listening to a half-hour news broadcast is not the same as reading the front section of a newspaper."

Q15 Some students might recognize that a written report in the newspaper contains much more detail than a typical spoken report on the radio. So, one way to make the timing about the same would be to edit each newspaper story down to the essential content. Other students might realize that a typical news broadcast reports on only a few "big" stories, whereas a newspaper tends to include major and minor stories. So, another option would be to read only, say, every other story in the newspaper. Lastly, some students might suggest getting a different newspaper that focuses only on major national stories and includes a lot of pictures, graphics, and/or advertising. For example, reading *USA Today* might be more equivalent to listening to a news broadcast than, say, reading the *New York Times.*

EXPLORE MORE

1. When the attributes are swapped, the slope represents the number of seconds per centimeter, or how long it takes to read a given amount of text. This is the reciprocal of the slopes used previously in this activity.

2. These questions give students mental exercise thinking about function scaling. Some students might be interested in actually collecting raw data, but encourage them to think through the situation first.

   If the columns were twice as wide, it would take you twice as long to read each line and you'd cover half the vertical distance. So, in general, this would halve the slope.

   If the columns were half as wide, it would take you half as long to read each line and you'd cover twice the vertical distance. So, in general, this would double the slope.

EXTENSIONS

1. Have students make two separate scatter plots for *VertDist1* versus *Time* and *VertDist2* versus *Time.* Without changing the data points, find ways to make the graphs look like you read faster than your partner. Find ways to make it look like your partner reads faster. Can you make it look like you both read at the same speed? Explain how to tell which person actually reads faster, no matter what the graphs look like.

   In step 14 of the activity, putting both *VertDist* attributes on the same graph means that one set of points always looks steeper than the other (unless the students read at the same speed). However, creating separate graphs allows you to rescale one graph's axes to give the appearance of steepness. The graphs that follow use different vertical scales to give the impression that the second graph is steeper. But, when you calculate the slopes (see the equations), the first graph is really steeper. This is a good opportunity

to discuss ways that data can be distorted, as is occasionally done in the media.
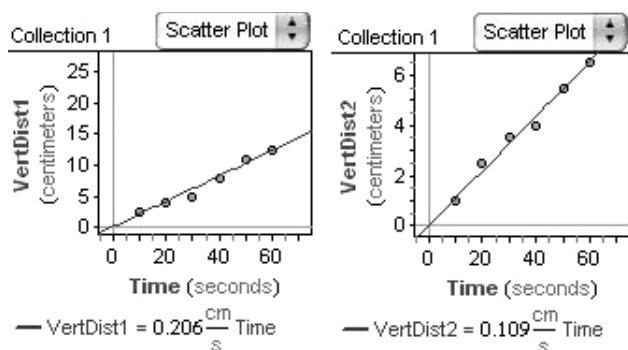
2. For students who are studying the least-squares line, have them make a graph of *VertDist* (1 or 2) versus *Time.* Add a least-squares line and lock the intercept at zero. Now drag any one point. Watch how coordinates of the data point change in the case table and how the value of the slope changes in the linear equation. Also, watch how the value of r^2 changes. Experiment to find the largest and smallest slopes you can get by dragging one point to different locations.

   Points can be dragged outside of the window into areas that are meaningless for this situation, such as the second, third, or fourth quadrants. If you restrict points to the first quadrant, the smallest slope will be close to 0. The largest slope depends on the data, but it will probably be smaller than 3.5. If you allow the points to be dragged anywhere, you can get just about any slope.

   Note: Dragging points is dangerous because you usually want to avoid inadvertently changing your data. Fortunately, you can always use **Undo** to make things right again.

SLOPE-INTERCEPT FORM (PAGE 11)

Activity Time: 30–50 minutes

Required Document: SlopeIntercept.ftm

Optional Document: SlopeInterceptPREMADE.ftm

Fathom Prerequisites: Students should be able to

- Create attributes defined by formulas
- Make a scatter plot of two attributes

Fathom Skills: Students will learn to how to

- Create sliders
- Graph lines of fit (median-median lines)
- Duplicate graphs
- Link graphs' axes
- *Optional:* Add multiple attributes to an axis (see Explore More 1)
- *Optional:* Plot functions (see Extension 2)

Mathematics Prerequisites: Students should be able to find the slope of a line by counting rise over run or using two points in the slope formula.

Mathematics Skills: Students will learn how to transform bivariate data by manipulating a table of coordinates; predict the graphical and functional transformations that result from transforming data; and determine the effects of the parameters m (slope) and b (y-intercept) on the linear equation $y = mx + b$.

General Notes: You and your students will immediately notice that this activity is different from most Fathom activities. Instead of analyzing real-world data with variability, this activity uses data generated by a linear function. This allows you to use Fathom as a tool for algebraic manipulation.

In this activity, students will transform a set of linear data and see how the equation of the line of fit changes. In turn, students will learn how the parameters m (slope) and b (y-intercept) affect the graph of the linear equation $y = mx + b$. This gives students in-depth experience with the multiple representations of function: table, graph, and algebraic equation.

If you make extensive use of graphing calculators in your algebra class, the structure of this activity may seem familiar. Transforming the data by creating new attributes is very similar to creating lists on a graphing calculator. And using sliders to explore the effects of parameters is similar to widely used programs that explore function parameters. Fathom, however, adds a level of sophistication by allowing students to see either several graphs that each show one line or, by adding multiple attributes to an axis, one graph that shows several lines. For these reasons, you may find this Fathom activity to be a good supplement to your use of graphing calculators.

Be aware that there is a subtle distinction between transforming data and transforming a function that might confuse some students. For example, consider that you have data in the form (x, y) from the line $y = x$. When you transform the data into the form $(x, y + 5)$ by translating each point up 5 units, the line that fits is now $y = x + 5$. Some students may be puzzled because you add to the y-coordinate, yet the equation compensates by adding to the x-value. One explanation is that the equation must maintain the same equality as before the data were transformed, or $y = x$. So, if you add to the y-coordinate, the equation must add the same amount to the x-coordinate— substituting $(x, y + 5)$ into $y = x + 5$ gives $y + 5 = x + 5$, or $y = x$. Another explanation is that the equation must undo the transformation in order to be consistent with the original data. For this explanation, rewrite the transformed equation as $y - 5 = x$; when you substitute $(x, y + 5)$, you get $y + 5 - 5 = x$, or $y = x$. The latter explanation is more useful because it helps explain a wider array of function transformations. For example, in this linear scenario, it explains why data transformed to the form $(x + 3, y)$ are modeled by $y = x - 3$. In general, it explains why data in the form (x, y) from the function $y = f(x)$ can be transformed into $(x + h, y + k)$ and the function $y - k = f(x - h)$.

Please note that this activity as written is designed to teach linear equations in slope-intercept form *and* to teach students how to use several of Fathom's features, such as sliders and duplicate graphs. If your students are already familiar with Fathom, or you want to avoid the time required to teach the software to novice students, you may prefer to use the document **SlopeInterceptPREMADE.ftm,** which has the sliders and graphs prepared for you. That

way, students can complete the worksheet by answering only the questions beginning with Q2.

For students in Algebra 2 or Precalculus, you can use this activity as an introduction to function transformations. You can also change the formula for y in the case table and explore transformations of nonlinear functions (see Extension 2). However, if you want to do an entire activity on transformations of a variety of functions, see the activity Function Transformations in the Algebra 2 chapter of this book, page 38.

INVESTIGATE

Q1 The equation is $y = x$. *Note:* This step uses the median-median line for convenience. If your students are familiar with this line, you may want to discuss why it fits the points perfectly.

Q2 Each coordinate *yPLUSb* is 4 more than the corresponding coordinate *y*. Students will probably guess that this will make each point in the scatter plot move up 4 units.

Q3 Each point in the scatter plot of (*x, yPLUSb*) is 4 units up from the corresponding point in the scatter plot of (*x, y*).

Q4 The equation changes to $y = x + 4$, which students might explain as "4 was added to x in the equation."

Q5 The y-intercept is 4.

Q6 Students should highlight that the points move up (or down, for negative) the same amount as *b,* the equation is always in the form $y = x + b$, and the y-intercept is always the same as *b*.

Q7 a. $y = x + 2$

  b. $y = x - 3$ or $y = x + (-3)$

  c. $y = x + 20$

  For the graphs in parts a and b, you may want to have students verify their answers by adjusting slider *b*. For the graph in part b, students will also need to adjust the domain and range of the graph's axes. The value of *b* for part c is purposefully beyond the range of the slider; this will force students to extend the concept of y-intercepts beyond what they can readily graph.

Q8 Students should highlight that the points are *m* times as far away from the *x*-axis and the equation is always in the form $y = m \cdot x$.

Some students may explain the effects of *m* on the scatter plot by describing special ranges of values, such as values greater than 1, values between 0 and 1, and negative values. Other students might describe how the slope of the line changes as *m* changes from $+10$ to -10 (from steep going up, to not-so-steep going up, to horizontal, to not-so-steep going down, to steep going down) or notice that *m* rotates the line.

Because students just finished examining the *y*-intercept, some might notice that the *y*-intercept is always 0 in these cases.

Q9 Answers will vary depending on how much experience students have with slope. If you encourage students to think about the formula for slope (or the rise-over-run definition), then they should be able to recognize that the slope is always the same as *m*.

Q10 a. $y = 0.25x$

b. $y = -3x$

c. $y = 15x$

Note: Similar to the graph in Q7c, the graph in Q10c intentionally uses a value that is beyond the range of the slider.

Q11 a. $y = 2x + 4$

b. $y = -1x - 5$

c. $y = 0.75x - 9$

d. $y = 12x$

e. $y = 3.5x + 2$

Q12 The linear equation $y = mx + b$ is called slope-intercept form because the value of *m* tells you the slope of the line and the value of *b* tells you the *y*-intercept.

EXPLORE MORE

1. The lines are *parallel* because they have the same slope.

- mTIMESy = 1.50x
- mTIMESyPLUSb = 1.50x + 3.0

○ mTIMESy   ■ mTIMESyPLUSb
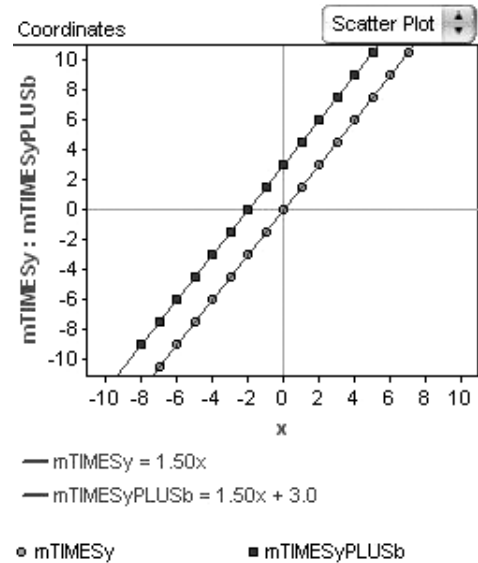
2. When using *m* and *b* with the *x*-coordinates, *b* relates to the *x*-intercept and *m* relates to the reciprocal of the slope. Hence, the equation for (*mTIMESxPLUSb, y*) is $y = \frac{x}{m} - b$.

EXTENSIONS

1. If your students have already learned about transformations, review the four fundamental types of transformations: translations, reflections, rotations, and dilations (sometimes called stretches or shrinks). Ask, "What type of transformation happens to the graph of the points (*x, y*) when you add *b* to *y*?," "What type of transformation happens when you multiply *m* times *y*?," and "Can you find a way to change the *y*-coordinates to create the remaining types of transformations?"

Changing the coordinates from (*x, y*) to (*x, y + b*) generally creates a vertical translation, although in the linear case it may also look like a horizontal translation. Changing the coordinates from (*x, y*) to (*x, m · y*) generally creates a vertical dilation, although in the linear case it may look like a rotation to some students and when $m = -1$ it is also a reflection across the *x*-axis. Here are some other transformations that

students can easily discover within the confines of this activity: $(x + b, y)$ creates a horizontal translation; $(m \cdot x, y)$ creates a horizontal dilation; $(x, -y)$ creates a reflection across the x-axis; $(-x, y)$ creates a reflection across the y-axis; (y, x) creates a reflection across the line $y = x$; $(-x, -y)$ creates a 180° rotation about the origin; $(-y, x)$ creates a 90° counterclockwise rotation about the origin; and $(y, -x)$ creates a 90° clockwise rotation about the origin.

2. You or your students may notice that the values for y in the case table are gray. That's because the y-coordinates are defined by the formula $y = x$. Students in Algebra 2 or Precalculus, in particular, can edit the formula to $y = x^2$ or $y = |x|$, for example, and explore the effects of changing the coordinates. In what ways are the results the same? In what ways are they different?

   Students should find that adding to the x- or y-coordinates creates a horizontal or vertical translation; multiplying the x- or y-coordinates creates a horizontal or vertical dilation; and negating the x- or y-coordinates creates a reflection across the y- or x-axis.

   If students want to explore the effects on the functions that fit the points, they should turn off the median-median line and choose **Plot Function** from the **Graph** menu. This will be more challenging, because students need to have an idea of what function to graph.

   By using nonlinear functions, students should begin to see that the equations that fit the points need to undo the transformations. Hence, the points $(ax + h, by + k)$ are fitted by the function $\frac{y - k}{b} = f\left(\frac{x - h}{a}\right)$, or $y = b \cdot f\left(\frac{x - h}{a}\right) + k$.

   If you want to pursue this extended extension with Algebra 2 or Precalculus students, see the activity Function Transformations in the Algebra 2 chapter of this book, page 38.

THE OCEAN TRAIN (PAGE 15)

Activity Time: 40–55 minutes

Materials:

- One copy of the train schedule (page 18) per group. Note that page 18 has two copies of the schedule.

Optional Document: OceanTrain.ftm

Fathom Prerequisites: Students should be able to

- Create a collection by entering data into a case table
- Create attributes and define them with formulas
- Make a scatter plot

Fathom Skills: Students will learn how to

- Work with units
- Manipulate data by creating and combining attributes
- Graph lines of fit (movable lines)

Mathematics Prerequisites: Students should be able to make a scatter plot of points; find the slope of a line by counting rise over run or using two points in the slope formula; and interpret slope-intercept form of a linear equation.

Mathematics Skills: Students will learn how to accurately read a nonstandard table of data; find a slope-intercept linear equation to model data that are roughly linear; interpret slope as a rate or speed; compare slopes based on their graphical and numerical representations; and manipulate and transform data so that they can be best modeled by a linear function.

General Notes: Like Reading the News, this activity is about slope and equations of lines. Students graph data that are roughly linear and model them with linear equations. They must be able to interpret the slope as a rate or speed, as is appropriate for the context of the data. However, there are two key differences. First, the data in this activity do not logically start at the origin. So, students must work with an equation in slope-intercept form and interpret the meaning of the intercept. Second, the data require some manipulation and transformation before they have a good linear pattern.

Here's a synopsis of the activity: Students begin by reading and interpreting a schedule for the Ocean, a Canadian passenger train. The schedule is not a standard "textbook" table, so students have to use a lot of problem solving to fully understand all of the data. Then they enter the data into Fathom and make scatter plots of distance versus time. However, because the data have many layers (time zones, days, hours, and minutes), students need to define new attributes and combine attributes to get an acceptable representation of time. After making the data look linear, students use movable lines to answer questions about the train's speed.

For all of the graphs in this activity, slope is the same as speed, as long as the right attributes are on the right axes. You might get in the habit of pointing to the vertical axis, saying "per," and pointing to the horizontal. You could also explain that slope is rise (vertical) over run (horizontal), and speed is distance over time, so kilometers goes on the vertical axis and time on the horizontal axis.

If your students have done the activity Slope-Intercept Form, they already know how to define attributes with formulas. In this activity, however, students will gain a deeper appreciation of formulas. Here, they combine attributes to accomplish something useful—to convert days, hours, and minutes into a useful measure of time. The idea that transforming data creates changes in the graph of the data is still visible, as the points gradually become more and more linear.

The portions of the activity that require students to manipulate the data are purposefully left open-ended, as opportunities for discovery learning. For example, question Q8 and step 4 ask students to find a way to make the data look like one line without suggesting what attributes and formulas to use. This allows students to find alternative, even unique, solutions that they may not have found if the worksheet had told them to simply add 1 day (24 hours) to each *Hour* after midnight. The discovery approach may be challenging for many students, and some may not "fix" the problem on the first try, yet Fathom allows students to easily change, correct, or undo their wrong steps.

The open-ended approach may be frustrating to some teachers, too. If you prefer to have a more guided activity,

you can periodically stop and have a class discussion to help students realize the optimal solutions. For Q8, one solution is to first create an attribute that measures days elapsed and add that to *Hour*. For step 5, one solution is to add *Minutes* to the sum of days elapsed and *Hour*. For Explore More 1, one solution is to create an attribute that adjusts for time zones and add that to the sum of days elapsed, *Hour,* and *Minutes.*

This activity will also help students see how Fathom works with units, performing the necessary unit conversions when operating with different units. For example, if you define a new attribute as the sum of an attribute with hours as the unit and another with minutes as the unit, the new attribute will have minutes as the fundamental unit (although, if the units are not specified, some of the cases may be displayed in hours). If students haven't had a lot of experience with units in Fathom, you might want to have them play around with unit conversions. For example, have students change the unit for the attribute *Hour* from h to min and watch how Fathom recalculates the values accordingly. Try changing the unit to days (d) or years (y), and then back to h again.

Some teachers may want to divide this activity over two classes. On the first day, students can read the schedule and enter data into Fathom. On the second day, they can begin analyzing the data with scatter plots and lines. Alternatively, if you can spare only one day, you can omit data entry and give students the pre-made document **OceanTrain.ftm.** This document has the schedule already entered, so you could skip steps 1–2 on the student worksheet.

UNDERSTAND THE DATA

Q1 Tuesday. Students should decode the "x2" below the Ocean train's name.

Q2 1:05 P.M. Students should look at the first column, which is for November to April.

Q3 304 km. Look in the KM column, which measures distance from Halifax.

Q4 Rimouski, at 12:23 A.M. (Eastern Time).

Note: Some students might notice that the schedule includes two time zones. So, it is also possible to say the answer is Amqui, at 12:01 A.M. Atlantic Time.

Q5 Causapscal. Students should follow the arrow and skip over all the cities from Gaspé through Nouvelle, which are cities for the Chaleur train.

Q6 461 km. Subtract their distances from Halifax.

INVESTIGATE

Note: As mentioned in the answer to Q4, some students may notice that the schedule gives times in two time zones. These same students may adjust each value of *Hour* to be consistent with one time zone or another when entering data into their case tables. This will create a few changes to the following answers.

Q7 There are two lines because the attribute *Hour* doesn't account for the fact that the train trip takes place on two days. The bottom-right points are the first day of travel; the upper-left points are the second day.

Q8 One solution is to create a new attribute called *Day* that gives the number of the day. Hence, Halifax to Mont-Joli would be 0 d and Rimouski to Montréal would be 1 d. Then create another new attribute, *DayHour,* that calculates the sum of *Day* and *Hour,* which effectively adds 24 hours to each time on the second day.

Note: Some students might want to use 1 d and 2 d to indicate the first day and second day, respectively. This is all right, though not optimal. In the long run, these students will still get a line with the same slope (speed), which is the fundamental question in this activity.

Note: Students who have adjusted *Hour* for the Atlantic Time zone should consider Amqui, Sayabec, and Mont-Joli as being part of the second day of travel.

Q9 Sample graph and equation:
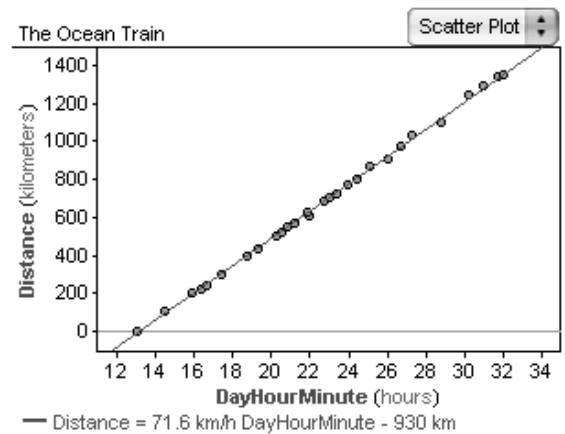
Q10 The slope of the line represents the train's average speed. Reasonable answers are between 70 and 75 kilometers per hour. Compared to automobile speeds, the speed may seem a little too slow, but then it does include stops at stations.

Note: Some students may not have a good conceptual understanding of speeds in kilometers per hour. If they comprehend miles per hour better, encourage them to convert units. For example, 70 km/h is about 43 mi/h.

Note: Students who have adjusted *Hour* for time zones may get a slope/speed as low as 65 km/h.

Q11 Reasonable answers are between −800 and −1000 km. In terms of the train schedule, the intercept shown above, (0, −930), means that if the train were to travel "backward" from its starting time of 1305 to 0000 (midnight) of that day, it would have gone backward about 930 km. Another way to think about this is that by not traveling between 0000 and 1305, the train lost 930 km that it could have traveled.

EXPLORE MORE

1. Students can either consider all times to be Atlantic Time, in which case 1 hour needs to be added to every time in Eastern Time, or consider all times to be Eastern Time, in which case 1 hour needs to be subtracted from every time in Atlantic Time. One way to do this is to manually change the values for *Hour*. If you prefer not to change the original data, then you

could create an attribute *Zone* that is 0 h (or −1 h) for all Atlantic Time stations and 1 h (or 0 h) for all Eastern Time stations. Then define an attribute *ZoneDayHourMin* that sums *Zone, Day, Hour,* and *Minute.* If students readjust the line of fit, they'll probably notice that the missing hour makes the slope decrease quite a lot.

The Ocean Train

— Distance = 67.2 km/h ZoneDayHourMinute − 870 km
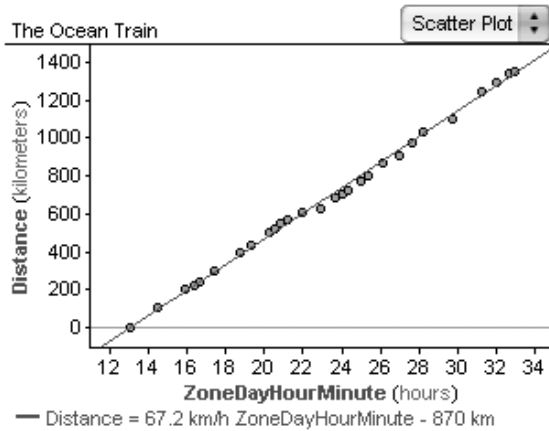
Note: If students seem interested in the results of progressively adjusting the attribute for time, you may want them to further consider this: How do you deal with the stations at which there is a layover (20 minutes at Moncton and 2 minutes at Matapédia)?

Note: Students who have adjusted *Hour* for time zones should not do Explore More 1 because they already have.

2. Answers will vary depending on the groups of points that students zoom in on. In general, steeper slopes represent times when the train traveled at a faster average speed, and shallower slopes represent slower times.

   If students have not had a lot of experience zooming in, either by dragging the axes or with the graph inspector, you may need to provide extra guidance with this item.

EXTENSIONS

1. Teach students about the *prev* function, or have students use Fathom Help to learn about it themselves. Use this function to define attributes that calculate the change in *Distances* and the change in *Time* (*ZoneDayHourMin*) between each station. Then define another attribute that calculates the speed between each pair of stations. Ask "Between which two stations does the Ocean go the fastest? The slowest?"

   This excerpt from the case table shows appropriate formulas and the results for the first five stations. The fastest speed is about 106 km/h between La Pocatière and Montmagny; the slowest speed is about 20 km/h between Campbellton and Matapédia. You might want to discuss possible reasons why the train can go so fast between some stations and so slowly between others.

ChangeKilo	ChangeTime	Speed
kilometers	hours	km/h
Distance − prev (Distance, 0 km)	ZoneDayHourMinute − prev (ZoneDayHourMinute, 0 hr)	ChangeKilo / ChangeTime
0 km	13.0833 h	0 km/h
103 km	1.41667 h	72.7059 km/h
97 km	1.41667 h	68.4706 km/h
27 km	0.416667 h	64.8 km/h
16 km	0.283333 h	56.4706 km/h

2. Have sudents redo the activity using the data in either **Trans-Siberian Railroad.ftm** or **CoastStarlight.ftm.** Both of these documents are located in the **Sample | Documents | Social Science** folder. Students could compare the trains using the questions in the activity.

OLYMPICS (PAGE 19)

Activity Time: 40–55 minutes

Required Document: Olympics.ftm

Fathom Prerequisites: Students should be able to

- Look at data with the collection inspector or a case table
- Make a scatter plot
- Graph multiple attributes on one axis
- Make a line of fit for a scatter plot
- *Optional:* Create new attributes (see Extension 3)
- *Optional:* Make a one-variable graph (dot plot, box plot, histogram; see Extension 3)
- *Optional:* Create a summary table (see Extension 3)

Fathom Skills: Students will learn how to

- *Optional:* Make a residual plot (see Explore More 1)
- *Optional:* Show and interpret residual squares (see Extension 1)

Mathematics Prerequisites: Students should be able to make a scatter plot of points; find a linear equation to model data that are roughly linear; and interpret slope-intercept form of a linear equation.

Mathematics Skills: Students will learn how to use scatter plots to compare the linear patterns of different data sets and approximate the intersection of two lines from a graph (i.e., solve a system of two linear equations graphically).

General Notes: This activity is intended to reinforce and extend the mathematics skills that students have learned in the previous three Algebra 1 activities. First, students use scatter plots and lines of fit to describe the pattern of Olympic results for the 100-meter dash over time, similar to the use of scatter plots and lines of fit in Reading the News and The Ocean Train. Students are instructed to identify the slope and intercept of the line from the equation, as practiced in Slope-Intercept Form, but students are also required to interpret the slope and intercept in terms of the real-world situation. Then they extend their learning by comparing systems of lines for the 100-meter dash and the 200-meter dash, and for the men's

and women's 100-meter dash. The former system is inconsistent (parallel), but the latter system allows students to determine when women Olympians might run as fast as men. (See Extension 3 for ways to work with one-variable statistics, as was done in Mammals' Brains.)

The structure of this activity is designed to model the scientific method. Mathematicians, scientists, and statisticians rarely have a step-by-step process to lead them to conclusions. Instead, they begin with a question or an observation about the way things seem to work and make conjectures about what they think is true; then they gather data that might support (or refute) the conjectures, analyze the data, and revisit their conjectures. This activity encourages students to make conjectures about the data before they look at it. Students are encouraged to use everything they know (including their understanding of athletics, genetics, and physiological differences between men and women) to make conjectures about how the Olympic data will look, change, and compare. Then, as they do their step-by-step analysis in Fathom, they are encouraged to reflect on their conjectures and see whether they were right or wrong. For a more open-ended activity, you might give students only the first page of the worksheet (the Make a Conjecture questions Q1–Q5) and then challenge them to use Fathom to support their conjectures without step-by-step instructions.

Before beginning to analyze the Olympic results in Fathom, you may want to discuss with students what it means for one gold-medal time or distance to be better than another. In the case of timed events (e.g., 100-meter dash), improvement is shown by shorter times; on the contrary, for distance-measured events (e.g., discus throw), improvement is shown by longer distances. So, although "improvement" is frequently equated with something increasing, students will need to look for data that either increase or decrease, depending on the event.

A technical note about the Olympic Games: The worksheet mentions that the Olympic Games have been held every four years since 1896. This is not entirely true for the Winter Olympics. From 1896 to 1992, the Winter and Summer Olympic Games were held in the same calendar year. However, beginning in 1994, Winter and Summer were staggered every two years. So now, although a

complete cycle of Olympic Games occurs every four years, there is a set of games every two calendar years.

If you would like to use this activity with Algebra 2 or Precalculus students, be sure to have them try nonlinear functions that could better model the Olympic data. That is, winning times cannot continue to decrease to zero and winning distances cannot continue to increase to infinity. Hence, curved functions with long-run limits would be more appropriate models. (See Extension 2.)

MAKE A CONJECTURE

Q1 Answers will vary depending on estimation skills and experience with track-and-field events. As a guide, the actual Olympic data range from 9.84 to 12 seconds.

Q2 Students will likely guess that the time has gotten shorter. Explanations may include improvements in training techniques, improvements in equipment (such as better shoes), athletes' desires to continually outperform records, or more rigorous selection of competitors.

Q3 Students will likely guess that the 200-meter results will be twice the 100-meter results.

Q4 Students will likely guess that the men's results are better than the women's results. Explanations may include that men are (stereotypically) stronger and can run faster than women, or that men generally have longer legs and can take fewer, bigger strides. They may also cite experience from physical education classes, where boys are likely to have run faster than girls. Students' answers to "How much better?" will rely on their estimation skills and frames of reference. If students have done track-and-field events in physical education class or after-school sports, they might cite differences anywhere from 1 to 5 seconds based on actual experience. The differences in the Olympic data actually range from 0.62 to 1.6 seconds.

Q5 Some students might guess that the differences are getting smaller because female athletes are continually becoming stronger and faster; they might even guess that the gold-medal times will eventually be the same for both genders. Other students might argue that the differences are staying the same because physiological

differences between men and women (bone structure, musculature, etc.) mean that men will always be a little faster.

INVESTIGATE

3. You may need to remind some students that *M_100Meter* versus *Year* means that *Year* goes on the horizontal axis and *M_100Meter* goes on the vertical axis.

Q6 The times have clearly gotten shorter (or improved) over time. Here's a sample graph with a median-median line added:


$$M\_100Meter = (-0.01075\ s)Year + 31.33\ s$$
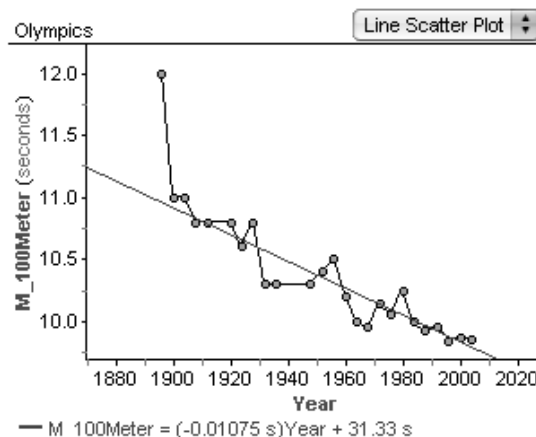
Q7 The greatest improvement was between 1896 and 1900, which is the longest downward segment. In general, downward segments (those with negative slope) represent improvements and upward segments (positive slope) represent setbacks.

Q8 The results for 1896, 1928, 1932, 1936, 1956, 1964, 1968, and 1980 are likely candidates. One method that students might use is to pick out the points that are farthest from the general linear trend.

Q9 For the median-median line, the slope means that the men's gold-medal times are decreasing by about 0.01 second per year. (*Note:* It's important that students realize the slope is defined per year even though the data are for every four years.) The *y*-intercept has no worthwhile meaning, although some students might try explaining it as the gold-medal time had the Olympics started in year 0.

Q10 2008 gives 9.75 seconds; 2012 gives 9.70 seconds. Students can answer this question either by substituting values into the linear equation and evaluating by hand or calculator, or by tracing the line of fit in Fathom.

Q11 The scatter plot shows that, for either men or women, every 200-meter point is about twice as high as the corresponding 100-meter point. Hence, the 200-meter results are about twice the 100-meter results. To see this clearly, students *must* adjust the vertical axis to begin at zero.
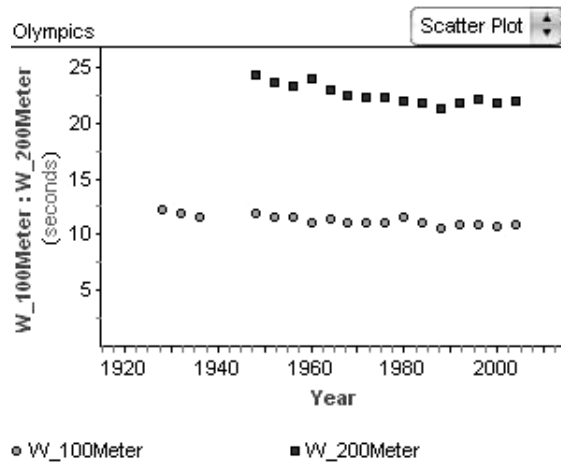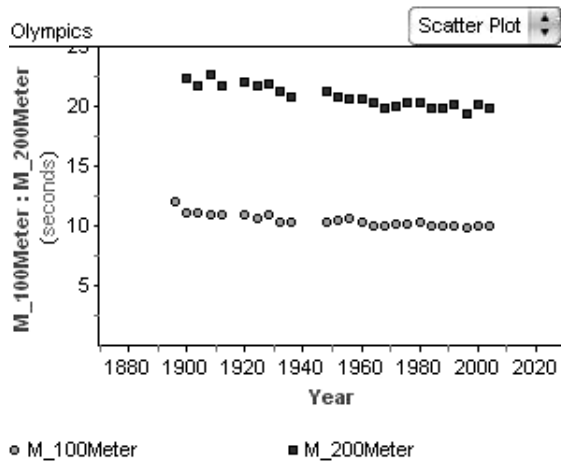
Q12 The graph supports the prediction that the men's results are better (smaller) than the women's results. For the answer to "How much better?" students might cite actual differences (0.62 to 1.6 seconds). Some students might try subtracting the two linear equations and state the difference as a function of time: *Difference* = -0.00512 s *Year* + 11.12 s.
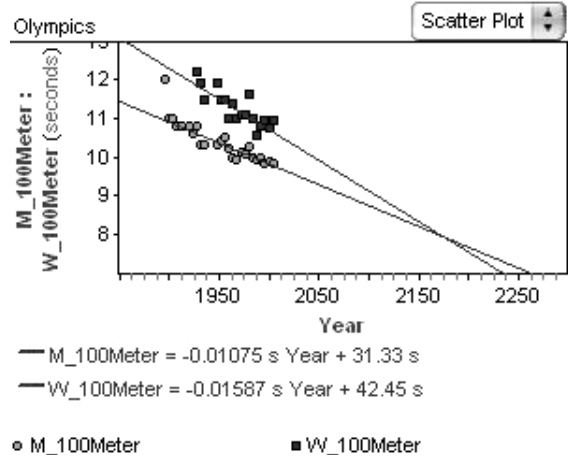

Q13 Yes, the difference is changing over time. You can tell because the lines are getting closer together.

Q14 Students find the intersection of their two lines of fit. For the median-median lines, the intersection is about (2175, 7.9), which means that in 2175 both men's and women's results for the 100-meter dash will be about 7.9 seconds. Because the lines do not fit the data very well and because the prediction is so far into the future, students should not feel confident about this answer. Furthermore, each line continues to decrease toward a winning time of 0 seconds—which is impossible—so the lines are an approximation at best.
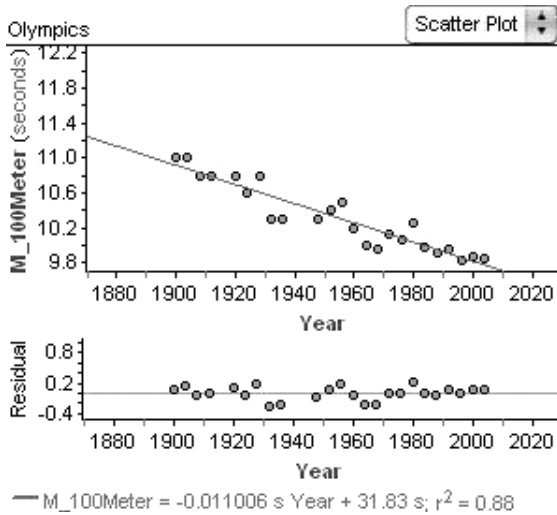
EXPLORE MORE

1. You identify outliers from the residual plot by identifying points that are farthest away from the horizontal zero line. Deleting outliers can have a dramatic affect on the line of fit, especially for a least-squares line. Because 1896 is the most obvious outlier, removing this one point significantly improves the lines of fit.

Here's the residual plot for the least-squares line:

— M_100Meter = -0.0131 s Year + 35.95 s; r² = 0.78
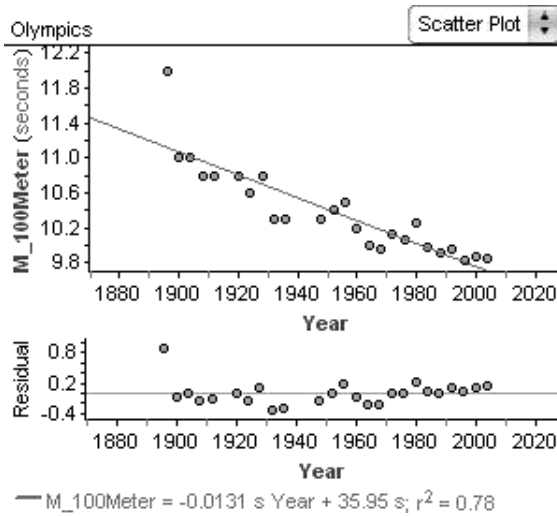
Here's the plot with the case for 1896 deleted:

— M_100Meter = -0.011006 s Year + 31.83 s; r² = 0.88

2. The discus results are the closest to showing no differ-ence between genders. Depending on the lines of fit used, someone could argue that a few events (e.g., marathon, high jump, long jump, and javelin) show a constant difference between the genders; that is, the lines of fit are roughly parallel.

EXTENSIONS

1. If you use Explore More 1 to investigate residual plots, a natural extension is to view the residual squares. To show the squares, choose **Show Squares** from the **Graph** menu. Ask students, "How are the squares

created?," "How do the squares relate to the residual plot?," and "How do the squares help you identify unusual points?"

— M_100Meter = -0.0131 s Year + 35.95 s; r² = 0.78
Sum of squares = 1.316
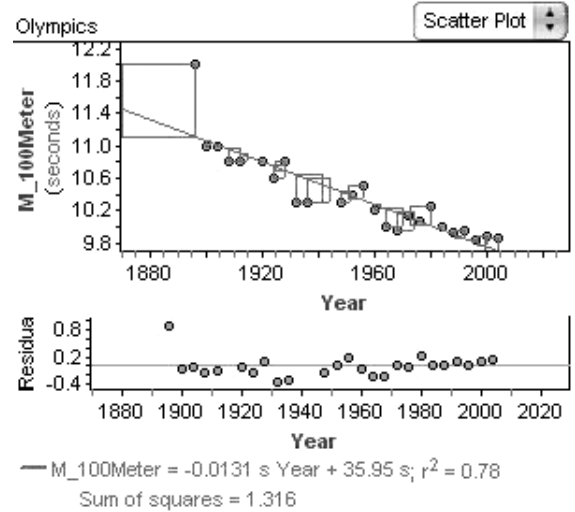
Note: If students have previously used least-squares regression in Fathom or on a graphing calculator, you can explain that the least-squares line minimizes the area of the residual squares.

2. Lines of fit are convenient but they are not always the best way to model data. Within the context of the Olympic data, discuss ways in which lines are of lim-ited use. Ask, "What other mathematical models might be appropriate for describing the Olympic data?"

Students should realize that linear models do not make long-term sense for most of the Olympic data. Namely, if you extrapolate far enough on a line with a negative slope, the line crosses the horizontal axis and has a value of 0; or, if you extrapolate on a line with positive slope, it goes to infinity. For the Olympics, it doesn't make sense, say, for someone to be able to run the 100-meter dash in 0 seconds or for someone to be able to throw a discus infinitely far. Students might argue that mathematical curves, such as exponential functions, are more appropriate models. Students in Algebra 2 or Precalculus can be challenged to find and plot better models.

3. Step 5 and Q11 can be further analyzed by perform-ing one-variable statistics, similar to those done in Mammals' Brains. Have students create a new

attribute that calculates the ratio *200Meter/100Meter* for each year. Then use dot plots, box plots, histograms, and summary tables to analyze the central tendency and spread of the ratios. This will show that for either men or women the ratio of winning times has remained closely proportional to the ratio of distances, or 2 to 1.
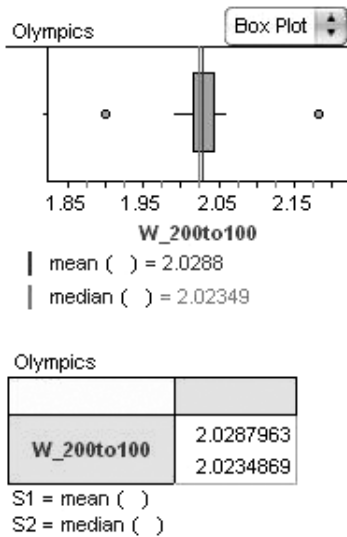
Then have students compare a "speed" event to an "endurance" event, such as the 100-meter dash to the 10,000-meter run (also called the 10K). Endurance races require that a runner maintain his or her stamina, so these athletes tend to run more slowly than sprinters. It is reasonable to assume that the ratio of 10K results to 100-meter results will be greater than the ratio of distance, or greater than 100 to 1. Using Fathom to analyze *M_10K/M_100Meter* gives a mean ratio of 169.
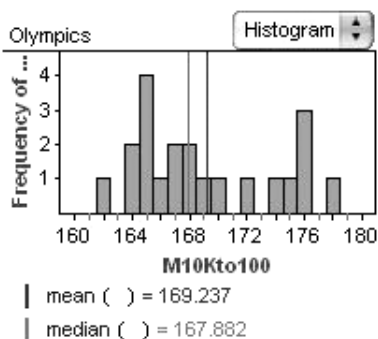
SEQUENCE OF SQUARES (PAGE 22)

Activity Time: 30–55 minutes

Fathom Prerequisites: Students should be able to

- Create a case table and collection
- Create attributes and define them with formulas

Fathom Skills: Students will learn how to

- Use special functions (*prev, next, caseIndex*) in formulas
- Add cases to a case table

Mathematics Prerequisites: Students should be able to recognize patterns in a sequence.

Mathematics Skills: Students will learn how to apply inductive reasoning; use technology to analyze a numeric pattern; and describe relationships *between* the square numbers. *Optional:* Students will recognize quadratic and other polynomial functions by analyzing finite differences (see Extension 1).

General Notes: This is perhaps the simplest, yet most open-ended, activity in this book. Students first use mental math to make a conjecture about a pattern in the sequence of square numbers. Then they use Fathom to analyze the pattern and confirm or refute their conjectures. The real task is being able to use Fathom to do *enough* calculations to support the conjecture. Along the way, students will gain a deeper understanding of square numbers and possibly get an introduction to quadratic functions.

Although this activity is highly appropriate for Algebra 1 students, it also has great extensions for students in Geometry, Algebra 2, or Precalculus. The activity itself is based on inductive reasoning—recognizing a pattern and extending it—but Explore More 1 has students generalize the pattern into proof. This is a great mental exercise for students at any level. In addition, Extension 1 uses the sequence to explore quadratic functions, other polynomial functions, and finite differences.

MAKE A CONJECTURE

Q1 It's expected that students will be able to use mental math to see that the difference between consecutive square numbers is a sequential odd number. Some students may further notice that the difference is twice the square root of the lower square, plus 1. That is, for example, the difference between 4^2 and 5^2 is $2(4) + 1$, or 9. If students don't make this observation in Q1, it's all right; hopefully they will notice it by the time they finish Q2.

INVESTIGATE

Q2 Students could simply type in square numbers for the cases, but this is time-consuming and fraught with human errors. Instead, students could define an attribute *Square* with the formula *caseIndex*^{2}. The function *caseIndex* refers to the case's row number. Students can then choose **New Cases** from the **Collection** menu to add as many square numbers as they desire.

There are many ways to calculate the differences between consecutive squares: *Square* − *prev*(*Square*) puts the difference beside the second square in each pair, so the sequence of odd numbers starts at 1; *next*(*Square*) − *Square* puts the difference beside the first square in the pair, so the sequence of odds starts at 3; and *caseIndex*^{2} − (*caseIndex* − 1)^2 and (*caseIndex* + 1)^2 – *caseIndex*^{2} give the same results as the *prev* and *next* formulas, respectively.

	Square	Difference
=	caseIndex^2	Square − prev (Square)
1	1	1
2	4	3
3	9	5
4	16	7
5	25	9
6	36	11

Q3 Hopefully, students made good conjectures in Q1 and the Fathom calculations support the conjecture. If not, they should now write a new, correct conjecture.

Q4 If students did not originally notice the "square root of the lower square plus 1" pattern, maybe they'll notice it here.

EXPLORE MORE

Here's a brief algebraic explanation:

$$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$$

Although this clearly illustrates the "square root of the lower square plus 1" pattern, some students may not recognize that it also illustrates the "odd number" pattern. You may need to explain that 2 times any number makes an even number, and then plus 1 makes an odd number.

Here's a geometric explanation:
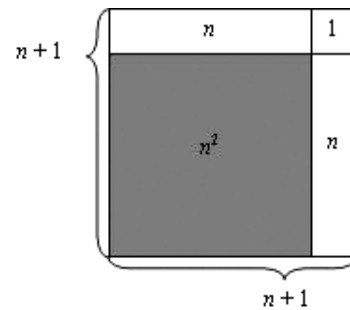
The gray square represents n^2 and the outer square represents $(n + 1)^2$. The difference between the two, the white region, is $2n + 1$.

EXTENSION

1. Have students add an attribute that calculates the differences of the differences. Ask, "What happens?" The second differences are constant.

Create another collection for cubes. Ask, "How many differences do you need to calculate before the differences are constant?" The answer is three sets of differences.

Challenge students to conjecture what will happen for *caseIndex*^{4}, *caseIndex*^{5}, and so on, and then test their conjectures. Also have them try any quadratic, cubic, quartic, or quintic function (for example, $3caseIndex^2 + caseIndex - 5$). Students should find

that the number of differences necessary to get constant values is always the same as the degree.

Now ask, "Assume you are given any sequence of numbers. How could you use what you've learned to determine whether the sequence is modeled by a polynomial function? How would you determine the degree of the function?"

THE CIRCUMFERENCE FUNCTION (PAGE 25)

Activity Time: 30–45 minutes

Materials:

- Many circular objects of different sizes
- Centimeter rulers or measuring tapes
- String

Fathom Prerequisites: none

Fathom Skills: Students will learn how to

- Create a collection by entering data into a case table
- Create attributes with formulas
- Create graphs (scatter plots)
- Graph lines of fit (movable lines)
- Use summary tables to calculate measures of center (mean)

Mathematics Prerequisites: Students should be able to make a scatter plot of points; find the slope of a line; and write the equation of a line in slope-intercept form.

Mathematics Skills: Students will learn how to approximate the value of π by analyzing the relationship between diameter and circumference; think about π as a functional relationship between diameter and circumference and as a ratio of circumference to diameter; find a direct-variation linear equation to model data that are roughly linear; and interpret the slope of a direct-variation linear equation as a constant ratio between the dependent and independent variables.

General Notes: Many geometry teachers already use an activity in which students measure the circumferences and diameters of circular objects and "discover" that the ratio is π. This is the same activity enhanced by an analysis in Fathom. By using Fathom to look at the data with tables, graphs, proportions, and summary statistics, students can get multiple representations of the same concept and gain a deeper understanding of π.

The activity opens by modeling the scientific method— challenging students to make a conjecture before they collect data. Because π is frequently taught in mathematics long before high school, many students will already know