To find the expected absolute value, find the mean of the absolute value of Location, weighted by the number of routes. So, add the absolute values of the products of the top and bottom rows of that table:

$$10 + 80 + 270 + 480 + 420 + 0 + 420 + 480 + 270 + 80 + 10 = 2520$$

That's the total distance for all the "walks," so divide by the number of walks, 1024. The expected value is $\frac{2520}{1024} = 2.461$.

EXPLORING SAMPLING (PAGE 81)

Activity Time: 30–45 minutes

Required Document: DeckOfCards.ftm

Fathom Prerequisites: Students should be able to

- Open files
- Create a case table
- Create a new attribute
- Use the formula editor

Fathom Skills: Students will learn how to

- Sample cases with and without replacement
- Change the properties of a sample, such as the sample size
- Use an inspector to create a measure
- Create a collection of measures

Mathematics Prerequisites: Students should be familiar with basic probability concepts.

Mathematics Skills: Students will create a simulation using sampling, understand the difference between sampling with and without replacement, and calculate an empirical probability from sampling data.

General Notes: Sampling is a central idea in statistics. This activity introduces the mechanics of sampling with Fathom and then uses sampling to investigate probability. Students explore the distinction between sampling with replacement and without it.

In question Q15, students are asked to find out how many pairs were drawn in all the samples from the entire class. You will need to facilitate this data collection in any way that is practical. We do this to increase the number of samples and thereby to decrease the sampling error of the probability we get.

The handout doesn't tell students how to make the formula for *pair*—the Boolean formula that assesses whether or not the two cards in the sample are a pair. Any formula that accomplishes the task is acceptable. Here are two possibilities:

Straightforward	$first(number) = last(number)$
Subtle	$uniqueValues(number) = 1$

Students are more likely to come up with the first formula, which works nicely. If they go on to the Explore More section, you should introduce them to the second formula. For the question on getting a pair in a five-card sample, the formula is $uniqueValues(number) = 4$. (The equals sign—as opposed to $\leq$—is correct. If $uniqueValues(number) = 3$, for example, that means the hand has two pair or three of a kind.)

MAKE A CONJECTURE

Q1 The teacher will write down 10 names, but the names may repeat. It is possible, but not likely, that the teacher will pick the same name 10 times in a row.

Q2 The teacher will write down 10 different names. It is not possible for the same student to be picked twice, because the names are not replaced after being picked.

Q3 If the teacher is sampling with replacement, she can theoretically continue picking students forever. If she is sampling without replacement, she can pick only 30 names.

EXPERIMENT

Q4 There are 10 names in the case table. It is sampling with replacement. This is evident because there are only 8 names, but 10 names in the sample. Some are repeating.

Q5 Each time the **Sample More Cases** button is clicked, a new set of 5 names appears in the case table. They may repeat.

Q6 Student results will vary. You can gather results from the class to show the spread of results.

Q7 No, the same name will never appear twice in the sample. If the sample size is changed to 8, all 8 names will come up every time (but the order will change).

Q8 All 8 names come up every time.

Q9 When **Sample More Cases** is clicked, the inspector changes the number of cases to 8. This is the maximum number of cases for sampling without replacement.

Q10 There are 52 cases in the collection. The attributes are *suit, number,* and *name.* The collection represents a deck of cards.

Q11 Clicking **Sample More Cases** brings up 10 new cards. Cards may repeat.

INVESTIGATE

Q12 One possibility is $first(number) = last(number)$. Another possibility is $uniqueValues(number) = 1$.

Q13 This collection has attributes *cardSize* and *pair.* There should be 5 cases if the sample size is set to its default of 5. Have students click the arrow to view different cases.

Q14 Sample answer: 15 cases true, 185 false. Students can use the graph to find the number of trues and verify that the bar counts add up to 200.

Q15 Make sure that you have recorded class values before moving on. The next part of the activity will wipe out the previous data.

Q16 Hopefully, there will be fewer trues—perhaps 12 out of 200.

Q17 Make sure students know that empirical probability is the same as experimental probability. Use the class results to calculate these answers. Sample answer: The probability would be $\frac{15}{200}$, or 0.08, for the pair with replacement, and $\frac{12}{200}$, or 0.06, for the pair without replacement.

EXPLORE MORE

1. You can have a more general discussion of the principles if students have not studied probability. Think about only the second card. With replacement, the chances are $\frac{4}{52}$ that you will draw a card of the same number. Without replacement, the probability is only $\frac{3}{52}$.
2. Note that the value of *number* for an ace is 1. You need to test the first and the last (second) card in the sample to see whether it is an ace or a ten card. (Face cards have a value of ten.) One way to do this is to create new measures in the original collection with these formulas:

$$hasAce = (first(number) = 1) \text{ or } (last(number) = 1)$$

$$hasTen = (first(number) > 9) \text{ or } (last(number) > 9)$$

$$blackjack = hasAce \text{ and } hasTen$$

The theoretical probability for getting blackjack without replacement is

$$\frac{4}{52} \cdot \frac{16}{51} + \frac{16}{52} \cdot \frac{4}{52} \approx 0.048$$

With replacement, the probability is

$$\frac{4}{52} \cdot \frac{16}{52} + \frac{16}{52} \cdot \frac{4}{52} \approx 0.047$$

3. Change the number of cases for the sample collection to be 5. To get a formula for the pair, use $uniqueValues(number) = 4$. This will return the cases where there are only four unique numbers, indicating that two cards must match.

ROLLING DICE (PAGE 85)

Activity Time: 30–45 minutes

Required Document: RollingDice.ftm

Materials: None (you may want to introduce the activity using a 6-sided die)

Fathom Prerequisites: Students should be able to

- Open files
- Create a case table
- Create a histogram
- Use an inspector
- Make collection attributes
- Create formulas

Fathom Skills: Students will learn how to

- Sample until a condition is true
- Graph a mean value
- Create an attribute for a measures collection

Mathematics Prerequisites: Students should be familiar with basic probability concepts.

Mathematics Skills: Students will differentiate between mode and mean, find a mode from a histogram, and relate the probability of an event to the expected number of trials to get a success.

General Notes: This activity explores how many trials it will take to achieve a particular result.

If the probability of an event is p, the expected number of trials you need to get a success is $\frac{1}{p}$. In this case, the probability of rolling a 6 is $\frac{1}{6}$, so the expected number of trials is 6.

Sampling does not necessarily involve choosing the size of the sample ahead of time. However, if it does not, you must be specific about the conditions under which sampling ceases. In the case of Fathom, you can control sampling with a Boolean expression.