

# Exploring Sampling

In this activity, you'll learn about sampling with and without replacement. You'll use what you've learned to simulate the probability of drawing a pair.

## MAKE A CONJECTURE

Imagine that the teacher has a jar containing pieces of paper with the names of every student in the class. The teacher draws a name, writes it down, and puts the paper back in the jar. She does this 10 times. This is sampling *with replacement*, because the names are put back in the jar.

- Q1** How many names will the teacher write down? How many *different* names will the teacher write down? Is it possible that the teacher will draw the same name 10 times in a row?


Now the teacher draws 10 names but does not put the papers back in the jar. This is sampling *without replacement*.

- Q2** How many different names will the teacher write down? Is it possible for the same student to be picked twice?
- Q3** If there are 30 students in the class, how many names can the teacher draw before running out of names if she is sampling without replacement? Sampling with replacement?

## EXPERIMENT

Now you'll simulate this situation in Fathom.

1. Open a new Fathom document.
  2. Make a new case table and give it one attribute, *name*. Type in eight different names, including your own. The collection—a box of gold balls—appears.
  3. Select the collection and choose **Sample Cases** from the **Collection** menu. A new collection named Sample of Collection 1 appears. Make a case table for that collection.
- Q4** How many names are in the case table? Does it appear to be sampling with or without replacement? Explain.



Collection 1

	name
1	Heather
2	Christian
3	Elizabeth
4	Bill
5	Jonas
6	Rommel
7	Marina
8	Aneesa

4. Double-click the sample collection (the second box of balls) to show its inspector. Click the **Sample** tab at the far right. Set the number of cases to 5. This means that 5 names will be chosen.

5. Click the **Sample More Cases** button repeatedly.

**Q5** Describe what happens.

6. In the inspector, change the number from 5 to 3. Click **Sample More Cases** again. Count the number of times you have to click **Sample More Cases** until you see the same name appear twice in the same sample of 3.

**Q6** How many times did you have to sample? Try the experiment again. Was your result about the same?

Now you will sample *without replacement*.

7. In the inspector, uncheck the With Replacement box. Click the **Sample More Cases** button repeatedly.

**Q7** Will the same name ever appear twice in the same sample of 3? Explain. What do you think would happen if you changed the sample size to 8?

8. Check your prediction by changing the number from 3 to 8—the number of names in your collection. Click the **Sample More Cases** button repeatedly.

**Q8** What happens in the case table?

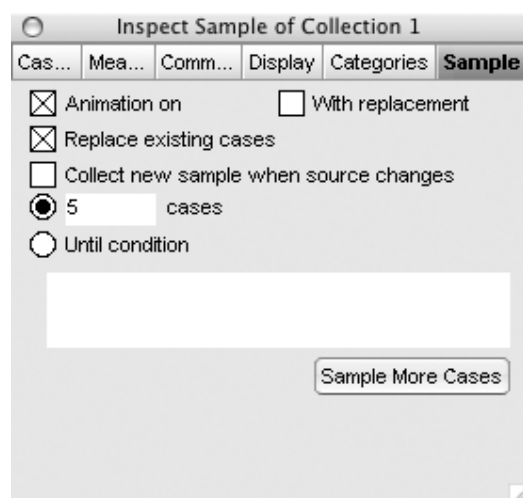
9. Now increase the number from 8 to 10. Click the **Sample More Cases** button repeatedly.

**Q9** Describe what happens. Watch the inspector closely.

Now you'll use what you've learned about sampling to simulate picking cards from a deck of cards.

10. Open the document **DeckOfCards.ftm**. Double-click the collection to show the inspector.

**Q10** How many cases are in the collection? What are the attributes?



11. Select the collection and choose **Sample Cases** from the **Collection** menu. A new collection appears, called Sample of Deck. Open the collection by dragging a corner of the icon.

**Q11** Click the **Sample More Cases** button several times. Describe what happens.

## INVESTIGATE

Now you will make a simulation to help you determine the chance of getting a pair when you draw two cards.

12. Change the number sampled from 10 (the default) to 2. Uncheck the Animation on box. When you click **Sample More Cases** (whether in the inspector or in the collection), you now get two cards.
13. To keep track of the number of pairs, you can record the results in a *measure*, or collection attribute. Click the **Measures** tab in the inspector to go to that panel. (You'll see a measure, *cardSize*, that controls how the cards look.) Click *<new>* to create a new measure. Call it *pair*.
14. Double-click the Formula cell to the right of *pair*. The formula editor appears. Give *pair* a formula that will be true if the two cards in the collection are a pair and false if they are not. A pair occurs when the values for *number* are the same. Three functions that might help are *first()*, *last()*, and *uniqueValues()*. They're all in the Statistical section in the Functions list.

**Q12** What is your formula?

Now you'll collect some different samples—different sets of two cards.

15. With the Sample of Deck collection selected, choose **Collect Measures** from the **Collection** menu. A new collection appears, called Measures from Sample of Deck. Show this collection's inspector and go to the **Cases** panel.
- Q13** What attributes does this collection have? How many cases does it have?
16. Click the **Collect Measures** tab at the far right. Uncheck the Animation on box. Check the Replace existing cases box so that you get a new set of samples. Increase the number of measures to 200 and click **Collect More Measures**. Be patient. You are collecting 200 samples of pairs of cards.
  17. With the Measures from Sample of Deck collection selected, drag a graph from the shelf. Go to the **Cases** panel of the inspector to see the attributes. Drag *pair* to the horizontal axis of your graph.

- Q14** How many of your 200 samples were pairs? How do you know? (You can click on the bar for “true” and see the number of “true” cases in the bottom-left corner of your Fathom window.)
- Q15** In the entire class, how many pairs were there? Out of how many samples?
- 18.** Show the inspector for the Sample of Deck collection (not the measures collection)—the one where you made the pairs. On its **Sample** panel, uncheck the With replacement box. You will now sample *without replacement*.
- 19.** Show the inspector for the Measures from Sample of Deck collection, and go to the **Collect Measures** panel. Make sure the Replace existing cases box is checked so that you will get 200 new samples. Click **Collect More Measures**. You’ll (slowly) get the data on 200 samples.
- Q16** How many of those samples were pairs? In the entire class, how many pairs were there? Out of how many samples?
- Q17** What is the empirical probability of getting a pair when drawing from a 52-card deck *with replacement*? What is the probability *without replacement*? Explain, in words, why one probability is greater than the other.

## EXPLORE MORE

1. Find the theoretical probabilities for drawing a pair from a 52-card deck with replacement and without replacement.
2. Modify the simulation to compute the probability of drawing two cards and have one be an ace and the other a king, queen, jack, or ten (getting a blackjack).
3. Modify the simulation to compute the probability of getting a pair in a *five*-card sample (a pair in poker). The *uniqueValues* function will help.