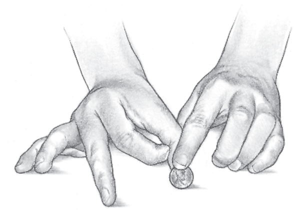
Constructing a Probability Model—Spinning Pennies

You will need
- one penny per student

For flipping a penny, heads and tails have the same probability. You might think that the same probability model is true for spinning a penny.

COLLECT DATA



1. Use one finger to hold your penny on edge on a flat surface, with the head right side up, facing you. Flick the penny with the index finger of your other hand so that it spins around many times on its edge. When it falls over, record whether it lands heads up or tails up.
2. Repeat until you have a total of 40 spins. Count the number of heads for your 40 spins.
3. Plot your value for the number of heads in 40 spins on a dot plot with the values of the other members in your class.

Q1 Are the data consistent with the model that heads and tails are equally likely outcomes, or do you think that the model can safely be rejected?

4. Pool your data for the number of heads with the rest of your class to get the total number of heads for your class. Record that value and the total number of spins for your class. Compute $\hat{p}$ for your class. Save this for later.

INVESTIGATE

Now you'll use Fathom to find out whether the "equally likely" probability model could generate results similar to the ones you observed.

To enter a formula, choose **Table | Show Formulas** and double-click in the formula cell.

5. In a new Fathom document, make a case table with the attribute *Face*. Add 40 new cases and name the collection Spins. Define *Face* with the formula randomPick("H","T"), which randomly selects heads or tails for each spin.

Spins

	Face
=	randomPick("H" , "T")
1	T
2	T

6. Show the inspector. On the **Measures** panel, define a measure as shown.

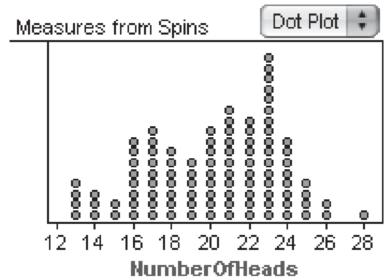
Inspect Spins

Cases | **Measures** | Comments | Display | Categories

Measure	Value	Formula
NumberofHeads	23	count(Face = "H")
<new>		

7. Select the Spins collection and choose **Collection | Collect Measures.** By default, Fathom collects five measures in a collection called Measures from Spins. Each measure is the result of spinning a penny 40 times.

8. Make a dot plot of *NumberOfHeads*.



To speed things up, uncheck Animation on.

9. Show the Measures from Spins inspector. On the **Collect Measures** panel, change the number of measures collected to 100 measures and check Replace existing cases. Click **Collect More Measures.**

Q2 How often did the number of heads from your spins appear in the measures? How many values were greater than your value?

Q3 Do you think the "equally likely" probability model applies to spinning pennies? Explain.

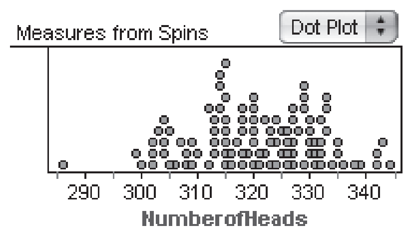
Now you'll look at possible values under the "equally likely" probability model for your whole class and see if that model holds.

You'll need to add the *class total of spins* − 40.

10. Add cases to the Spins collection so that the total number of cases in the collection is the same as the total number of spins your class collected.

Each measure will now be the result of spinning a penny the same number of times as your class did as a whole.

11. Select the Measures from Spins collection and choose **Collection | Collect More Measures.** Your dot plot will update. Here is a dot plot for a class with 16 students (640 spins).



Q4 How often did the number of heads from your class's spins appear in the measures? How many values were greater than your class's value?

Q5 Do you think the "equally likely" probability model applies to spinning pennies? Explain.

EXPLORE MORE

1. If getting heads or tails is equally likely, what is the probability of getting *exactly* 250 heads out of 500 spins? Use your simulation to estimate this probability.
2. Suppose you spin a penny and get 11 heads and 19 tails. Perform a simulation that tells you how often 11 (or fewer) heads could occur, assuming heads and tails are equally likely.
3. Suppose the actual probability of spinning heads is 0.4. Make a simulation to determine about how many spins are required before you can detect the difference between the 0.4 and 0.5 probability models.

Constructing a Probability Model—Spinning Pennies

Objectives

- Comparing actual results to a model to evaluate whether the observed results are consistent with the model
- Using simulation to estimate the probability of obtaining the observed results under the assumed model

Activity Time: 40–50 minutes

Setting: Paired/Individual Activity or Whole-Class Presentation (collect data, combine data as a class, then build simulation individually or as a class)

Optional Document: SpinSimulator.ftm (Explore More 3 solution)

Materials

- One penny per student (for U.S. pennies, those from the 1960s are better than newer ones, if you can get them)

Statistics Prerequisites

- Familiarity with sampling distributions
- Some familiarity with equally likely outcomes
- Definition of probability

Statistics Skills

- Probability simulation
- Working with the definition of probability and equally likely outcomes
- Comparing actual data to a hypothesized model
- Detecting differences between models

AP Course Topic Outline: Part III A, C, D

Fathom Prerequisites: Students should be able to make collections and graphs, add cases, and define attributes and measures.

Fathom Skills: Students use random generators to create a collection that represents a random sample, use Fathom to do simulations, and collect measures to compare models. *Optional:* Students collect measures from various size samples and use two parameters to test various models (Explore More 3).

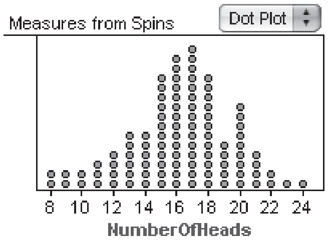
General Notes: This activity demonstrates to students the need for data when the hypothetical model (in this case, the "equally likely" principle) lets them down. The activity uses Fathom to repeatedly sample from a population in which spinning a coin has an equal probability of heads or tails. This allows students to focus on the underlying idea that testing a hypothesis involves comparing a particular result with a hypothetical sampling distribution.

Procedure: When U.S. pennies are spun rather than flipped, the data often support a model that has something other than 0.5 for the probability of getting heads. The probability of getting heads by spinning seems to be related to the year in which the pennies were minted. For example, 1990 pennies have a probability of around 0.4 for heads, whereas 1961 pennies have a probability of only about 0.1 for heads. So, in step 2 and especially in step 4, the proportion of heads will largely depend on the ages of your class's collection of pennies. However, most likely in step 4, the proportion of heads will be less than 0.5, which is not what students expect. Whether your students reject the model that spinning a penny is fair will largely depend on how far the proportion is from 0.5. If their answers change from Q3 to Q5, you have another opportunity to talk about the difference between small samples and large ones.

Steps 5–11 help students build a Fathom simulation to analyze the results, assuming that spinning a penny is fair. For steps 9 and 11, students should arrange their Fathom screen so that they can see the case table and dot plot change. It is recommended that students leave animation on in step 9 while collecting measures so that they can see the dot plot grow. They might need to turn animation off in step 11, depending on the speed of their computer.

COLLECT DATA AND INVESTIGATE

Q1–Q4 If students are using newer pennies ($p = 0.4$), they probably won't reject the equally likely model because with only 40 spins, it is quite plausible to get around 20 heads. For example, using the same type of simulation setup as the activity except using a model with $p = 0.4$:



Have students find the number of measures that are greater than or equal to the class's number of heads. They can do this using a summary table for the Measures from Spins collection.

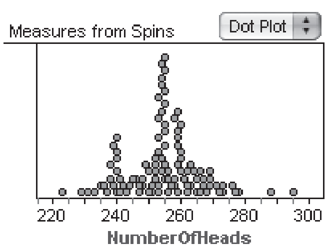
Measures from Spins

NumberOfHeads	8 0.17

S1 = count (NumberOfHeads = 20)
S2 = proportion (NumberOfHeads ≥ 20)

In this simulation, for 17 cases out of 100, the number of heads was 20 or more when a penny was spun 40 times.

Q5 It is likely that students will be able to reject the model. For example, in the sample data there were 640 spins. The equally likely model would predict 320 heads. However, out of 100 simulations of 640 spins (with $p = 0.4$), *none* of the 100 simulations approached 320.



DISCUSSION QUESTIONS

- What could explain the difference between the observed results and the equally likely probability model? Are you really convinced the model is wrong?
- Suppose you spin a coin four times and get no heads. What would you think about the coin at that stage? How many spins without any heads would you need before you became convinced something was "wrong"?

EXPLORE MORE

1. 0.0357. Keep 500 cases in the Spins collection and collect measures. Count how many are exactly 250. Values between 0.01 and 0.06 are acceptable.
2. 0.1002. Keep only 30 cases in the Spins collection and collect measures. Count how many are less than or equal to 11. Values between 0.04 and 0.15 are acceptable.
3. There are many ways to tackle this problem, and there is no exact answer. One approach is to replace the formula given in step 5 with randomPick("H","H","T","T","T") and run the simulation for different sample sizes. Compare the results of each sample size with the results for randomPick("H","T") of the same sample size. Look for the sample size at which the results are distinctly different.

   For another solution, see the document **SpinSimulator.ftm.** Here sliders control the sample size and the probability. With p set at 0.4, 100 measures were taken of 20 spins, then 40, then 60, and so on, until the number of measures greater than or equal to 0.5 was less than 5%. This happened between 60 and 80 spins in the simulation shown. To start this simulator from the beginning, select the measures collection and choose **Edit | Select All Cases,** then **Edit | Delete Cases.** Your measures collection will empty and you can start your own simulation by changing n or p and clicking **Collect More Measures** in the measures collection.