Teacher Notes

8.1b Constructing a Chart of Reasonably Likely Events

Fathom Skills

- Building a collection of measures from different populations (that is, a population collection that changes before each new set of measures is collected)
- Introducing true/false formulas (Boolean expressions)

Materials

- One copy per student of "A Complete Chart of Reasonably Likely Outcomes for $n = 40$" from page 126 of *Instructor's Guide, Volume 2* (optional)

Activity 8.1b is essential for your students to get an intuitive understanding of confidence intervals. This Fathom activity is presented in two parts.

Part A uses simulation to create a line-segment chart of reasonably likely outcomes for samples of size 40. (See Display 8.2 on page 472 of the student text or *Instructor's Guide, Volume 2,* page 110 or 127, for the actual chart of reasonably likely outcomes.) It parallels the activity as it is presented in the student text except that students make each segment on the chart with simulation rather than theoretical calculations. The simulation is fairly complex and may take longer than the activity in the student text. However, the simulation allows each individual student or group of students to make the entire chart themselves, thereby building a deeper understanding of how the chart is created.

Part B graphs the theoretical lower and upper bounds of the chart, using

$$p \pm 1.96\sqrt{\frac{p(1-p)}{n}}$$

This allows students to compare the simulated results with the theoretical results. It also gives students an electronic version of Display 8.2 that can be used throughout the chapter. (See the extensions for ways to make the chart change dynamically for different confidence levels and different sample sizes.)

If you do not have time to complete both parts of the Fathom activity, try using only Part A, only Part B, or the version in the student text followed by Part B of the Fathom activity.

What's Important Here

- Considering which possible population proportions could likely produce an observed sample proportion
- Developing an intuitive understanding of confidence intervals

Procedure

Part A: Reasonably Likely Outcomes by Simulation

It may be valuable to point out that this activity steps over one level of abstraction. Students will recall from other activities (for example, Activity 6.3a) that they first created a collection for the population, sampled from the population, then collected measures from the sample. In contrast, this activity starts with a collection that represents a randomly generated sample. Doing so makes the simulation a little less complicated.

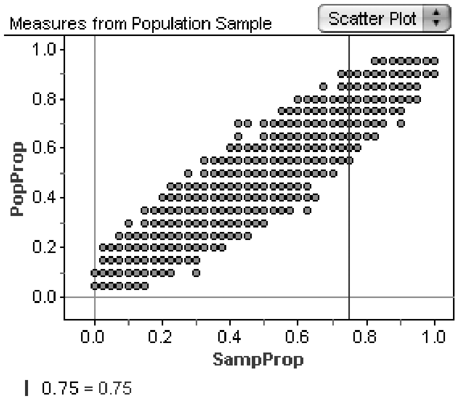
For step 5, the middle 95% of the sampling distribution for $p = 0.6$ is theoretically cut off by the two points

$$p \pm 1.96\sqrt{\frac{p(1-p)}{n}} = 0.6 \pm 1.96\sqrt{\frac{0.6(1-0.6)}{40}} \approx 0.6 \pm 0.152$$

So the horizontal "segment" of points should go from approximately 0.448 to 0.752. Simulated results will vary, of course.

The answers to step 6 will vary according to the results of Activity 8.1a. If your class's sample proportion from that activity was approximately between 0.448 and 0.752, then it is plausible that the true value of p might be 0.6.

Here is a sample scatterplot for step 9. If you wish, distribute copies of the completed, theoretical chart from *Instructor's Guide, Volume 2,* page 127. This chart, invented by Jim Swift, a high school teacher in British Columbia, is the key to understanding the confidence interval. Be sure students understand that the confidence intervals are read vertically and the reasonably likely outcomes are the horizontal line segments. If you plot the value of a sample proportion as a vertical line, for example, 0.75, it intersects the population proportions for which it is reasonably likely, 0.6 to 0.85. Students can use this Fathom document or the *Instructor's Guide* handout throughout this chapter.



Part B: Reasonably Likely Outcomes by Theoretical Values

For steps 3 and 4, students should adapt

$$p \pm 1.96\sqrt{\frac{p(1-p)}{n}} \text{ into } \mathbf{x - 1.96\sqrt{\frac{x(1-x)}{40}}} \text{ and } \mathbf{x + 1.96\sqrt{\frac{x(1-x)}{40}}}$$

(You enter the right side of the function only when using **Plot Function.**) Please note that these functions are technically invalid for population proportions less than 0.25 and greater than 0.75 because you shouldn't use the normal approximation when either np or $n(1-p)$ is less than 10. (Extension 1 attempts to address this issue.)

The art with step 9 of the student worksheet shows the result of Part B. With a sample proportion of $\hat{p} = 0.75$ (**pHat**), for example, you can adjust the slider **p** to approximate the bounds of the confidence interval at 0.614 and 0.886.

Discussion Question

- Explain how you use your scatterplot from Part A as a calculator to approximate the range of population proportions for which a given sample proportion is a reasonably likely outcome. Do the same for your graph from Part B.

Extensions

1. Students will need to plot these functions.

$$\textbf{PopProp} = \textbf{SampProp} - 1.96\sqrt{\frac{\textbf{SampProp}(1 - \textbf{SampProp})}{40}}$$

$$\textbf{PopProp} = \textbf{SampProp} + 1.96\sqrt{\frac{\textbf{SampProp}(1 - \textbf{SampProp})}{40}}$$

For population proportions less than 0.25, they should notice that many points fall outside the left bound. For proportions greater than 0.75, many points fall outside the right bound. As explained earlier, the functions are invalid for these values because either np or $n(1 - p)$ is less than 10. Students might want to explore the use of **binomialQuantile** to remedy this problem.